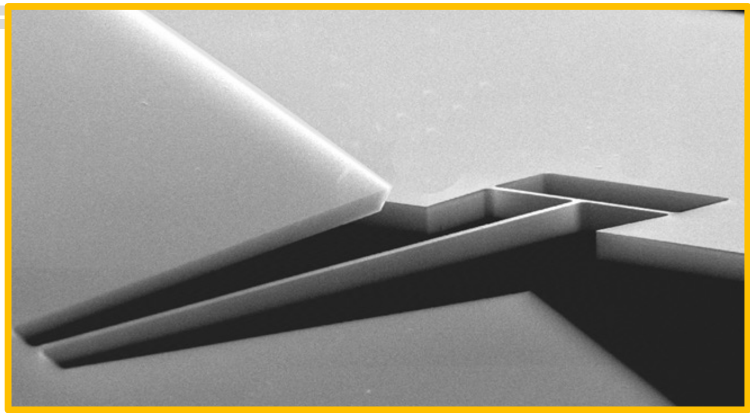
Problem 1. folded (serpentine) springs for torsion

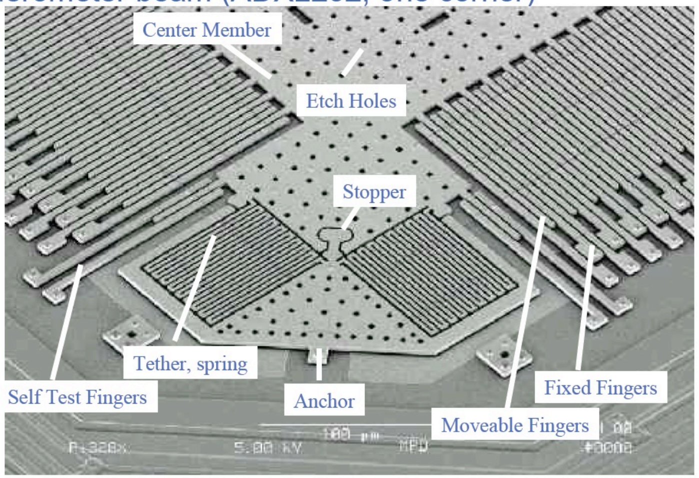
Serpentine springs
- take less area, and
- allow for more design freedom for different modes (but adds more challenges)

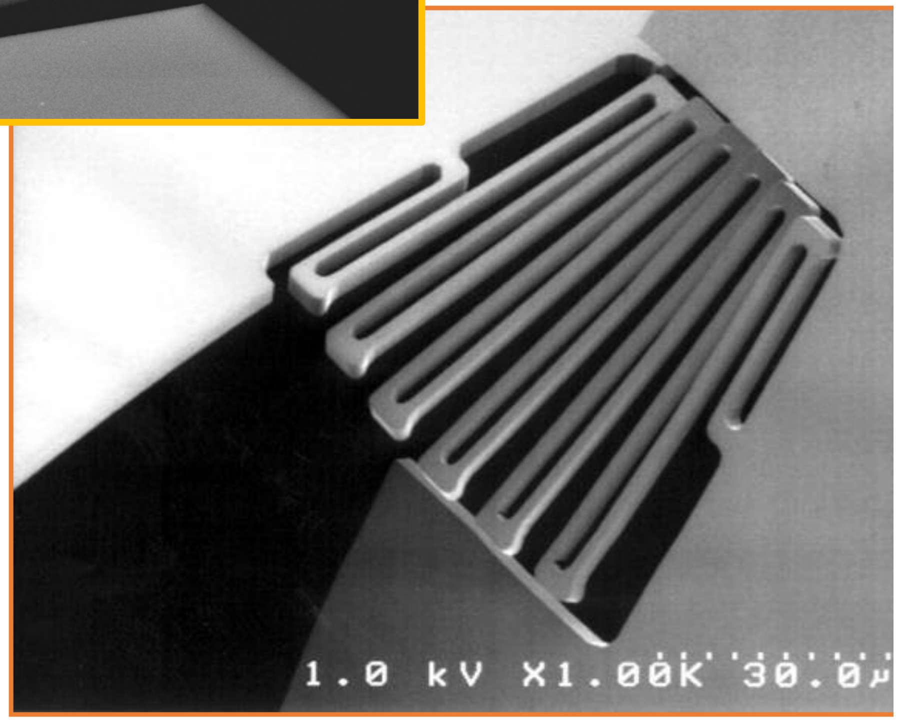
- Typically used either in bending or in torsion

- Need to consider bending stiffness on all axes!



Accelerometer beam (ADXL202, one corner)





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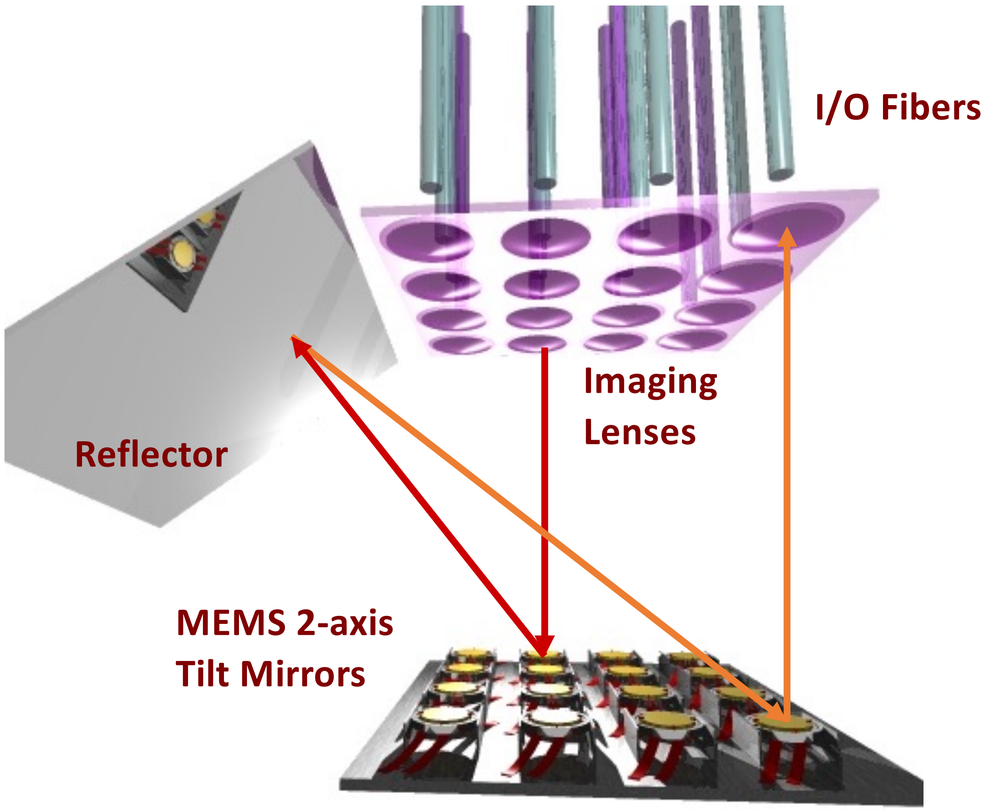
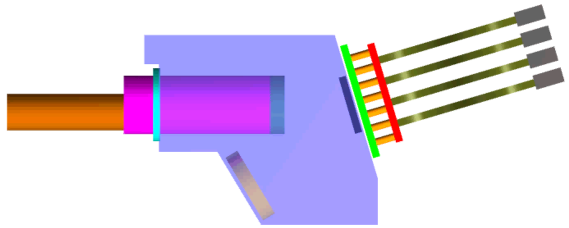
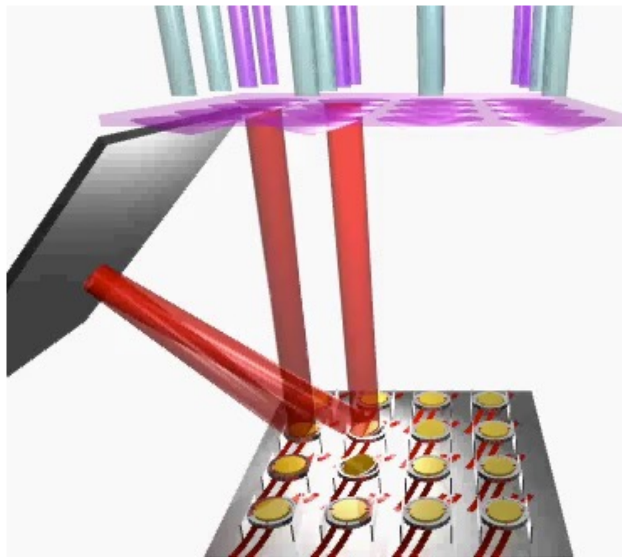
Example: MEMS mirror for Optical cross-connect

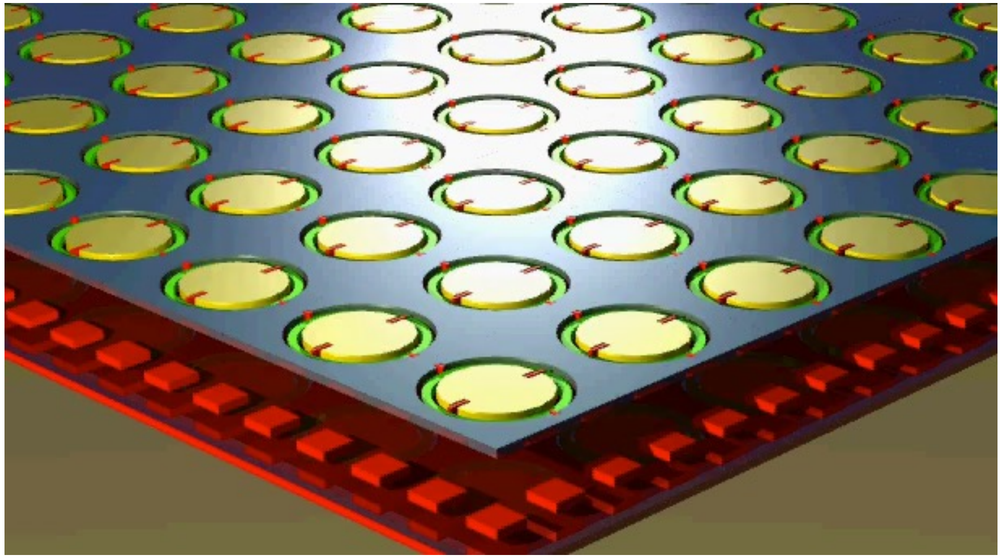




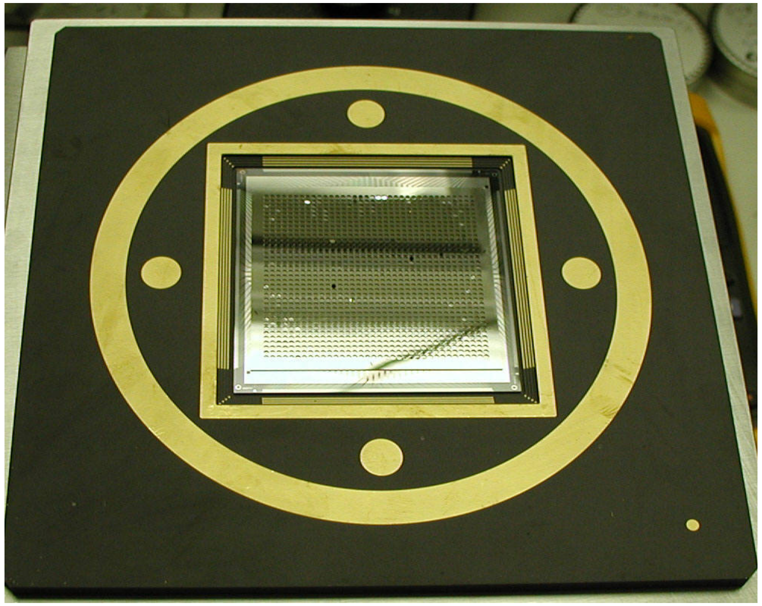


Beam scanning during connection setup.

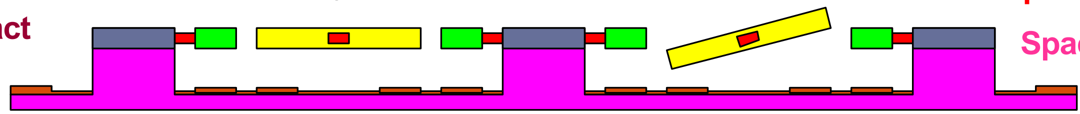
Single-Crystal Silicon (SOI), electrostatic drive
1296 Micromirror Array

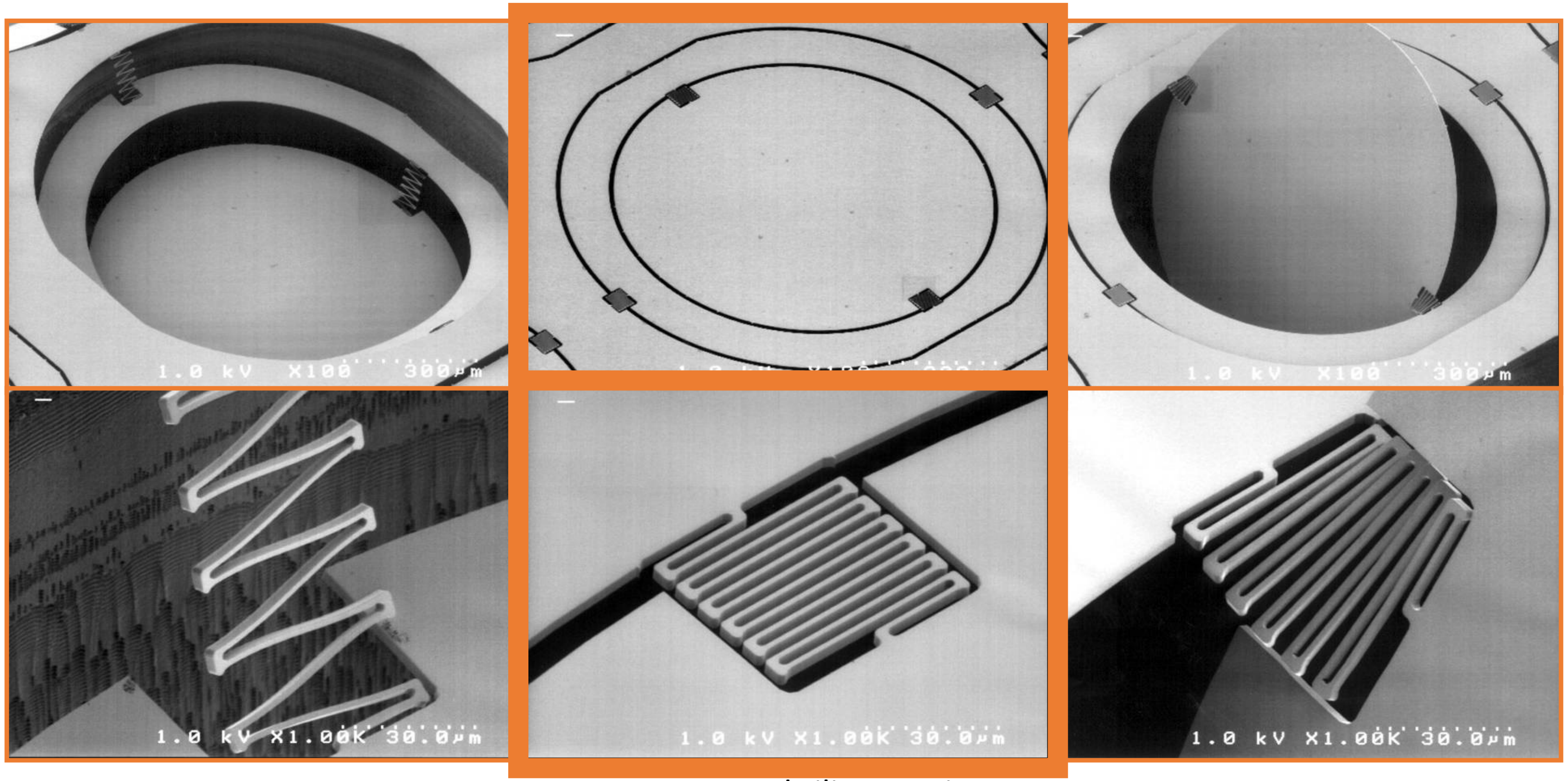


Voltage-actuated
mirror deflection



- 1296 mirror array
- SOI





unstressed silicon springs

Single crystal silicon, 5 μm thick. Springs are approximately 2 μm wide

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Bell Labs Innovations

Engineering Si springs for 1000 G (0.5 ms) shock tolerance: need resistance in 3 directions… all spring have the same torsional stiffness.

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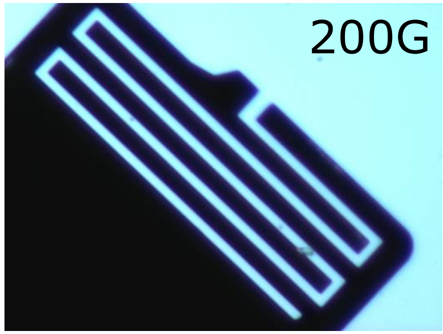
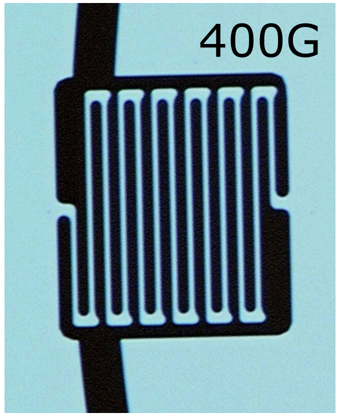
Bell Labs Innovations

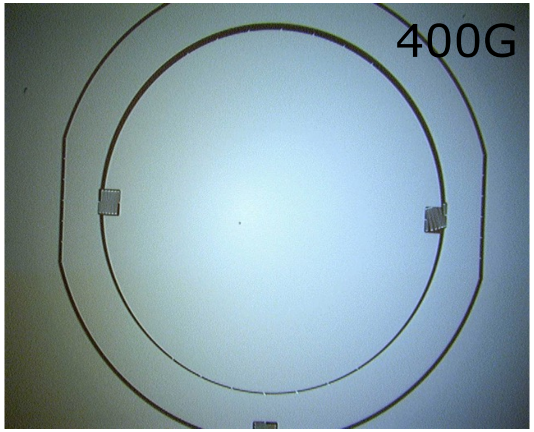


•First generation of mirrors failed at 200G

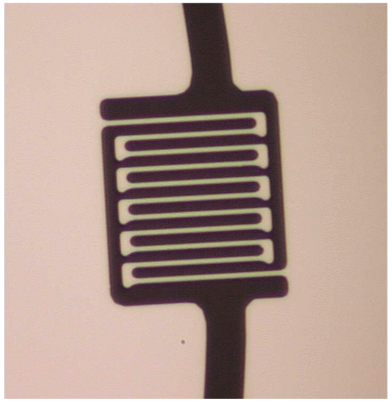
Failure mode: cracking of the spring at the 90^0 corner



•Next spring design incorporated widening at the 90^0 corners to suppress observed failure mode.



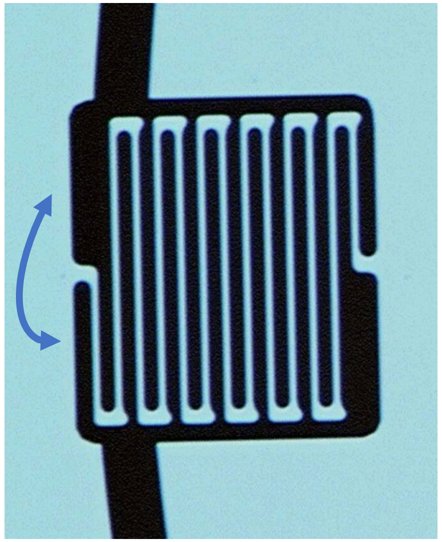
•Failure mode at 400G: due to soft lateral modes, mirror and/or gimbal slides under the gimbal and/or handle and gets stuck



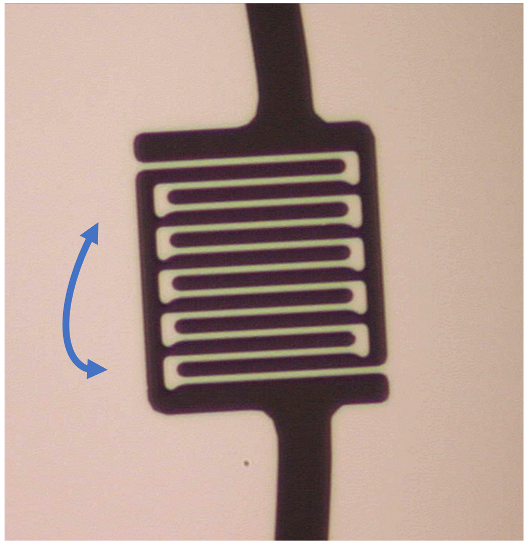
•Final design of 5um thick mirrors had 90^0 rotated serpentine springs for stiffer lateral modes. Shows excellent mechanical shock resistance.
•All tested mirrors remained functional after repeated 1000G mechanical shocks

Best beam design for one mode may be poor choice for other modes…

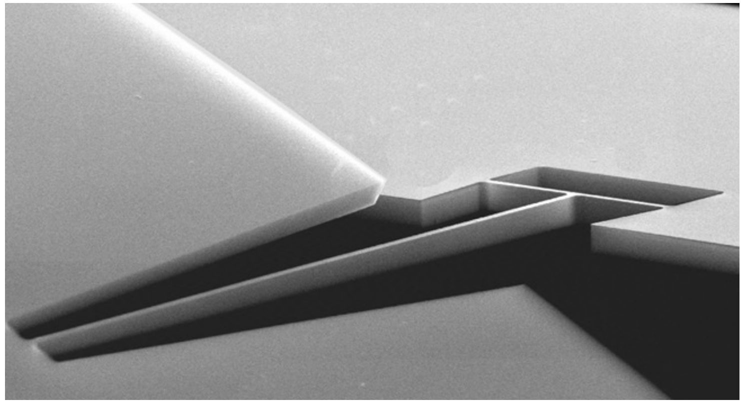
Different springs for torsional support

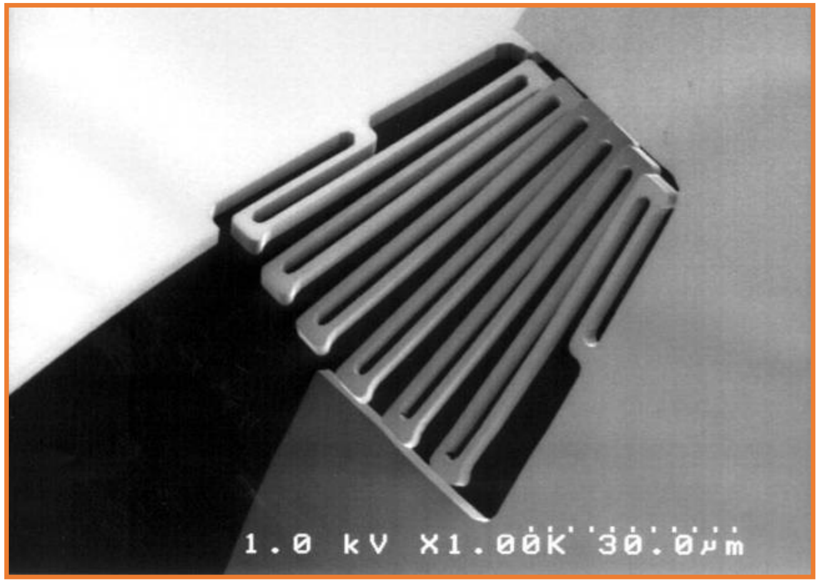


Beams in **bending**
(when mirror tilts)



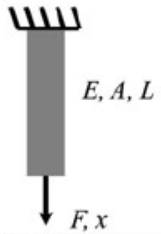
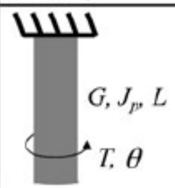
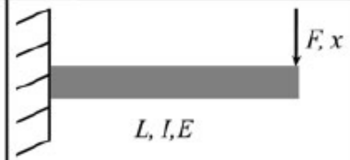
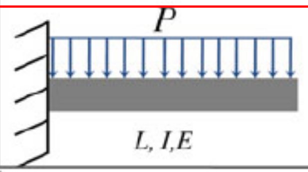
Beams in **torsion**
(when mirror tilts)





Spring constant in bending of a simple cantilever…

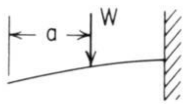
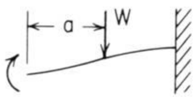
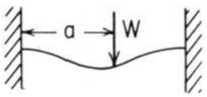
Table 4.1 The linear stiffness coefficient k of some of the common structure configurations in MEMS. The unit of k for all loading types is force/length. For wide beams ($b > 5h$) of cases 3–9, replace E with $E/(1-\nu^2)$, where ν is the Poisson's ratio. When calculating the natural frequency of a flexible structure alone, use an "effective" mass for the structure

1) Axially loaded bar		$k = \frac{EA}{L}$ E: Modulus of elasticity A: Area of cross section L: Length of bar
2) Rod under torque		$k = \frac{GJ_p}{L}$ J_p: Polar moment of inertia of the cross section G: Shear modulus L: Length of rod
3) Cantilever beam under point load at the tip		$k = \frac{3EI}{L^3}$ E: Modulus of elasticity I: Moment of inertia of the cross section L: Length of bar
4) Cantilever beam under uniformly distributed pressure		$k = \frac{8EI}{L^3}$

Younis, M. I. *MEMS Linear and Nonlinear Statics and Dynamics*. **20,** (Springer US, 2011)

Roarks' formula for stress and strain

Boundary conditions matter…

reference no.	Boundary values	Selected maximum values of moments and deformations
1a. Left end free, right end fixed (cantilever) 	$R_A = 0 \quad M_A = 0 \quad \theta_A = \frac{W(l-a)^2}{2EI}$ $y_A = \frac{-W}{6EI}(2l^3 - 3l^2a + a^3)$ $R_B = W \quad M_B = -W(l-a)$ $\theta_B = 0 \quad y_B = 0$	Max $M = M_B$; max possible value $= -Wl$ when $a = 0$ Max $\theta = \theta_A$; max possible value $= \frac{Wl^2}{2EI}$ when $a = 0$ Max $y = y_A$; max possible value $= \frac{-Wl^3}{3EI}$ when $a = 0$ $k = \frac{3EI}{l^3}$
b. Left end guided, right end fixed 	$R_A = 0 \quad M_A = \frac{W(l-a)^2}{2l} \quad \theta_A = 0$ $y_A = \frac{-W}{12EI}(l-a)^2(l+2a)$ $R_B = W \quad M_B = \frac{-W(l^2-a^2)}{2l}$ $\theta_B = 0 \quad y_B = 0$	Max $+M = M_A$; max possible value $= \frac{Wl}{2}$ when $a = 0$ Max $-M = M_B$; max possible value $= \frac{-Wl}{2}$ when $a = 0$ Max $y = y_A$; max possible value $= \frac{-Wl^3}{12EI}$ when $a = 0$ $k = \frac{12EI}{l^3}$
1d. Left end fixed, right end fixed 	$R_A = \frac{W}{l^3}(l-a)^2(l+2a)$ $M_A = \frac{-Wa}{l^2}(l-a)^2$ $\theta_A = 0 \quad y_A = 0$ $R_B = \frac{Wa^2}{l^3}(3l-2a)$ $M_B = \frac{-Wa^2}{l^2}(l-a)$ $\theta_B = 0 \quad y_B = 0$ Double clamped	Max $+M = \frac{2Wa^2}{l^3}(l-a)^2$ at $x = a$; max possible value $= \frac{Wl}{8}$ when $a = \frac{l}{2}$ Max $-M = M_A$ if $a < \frac{l}{2}$; max possible value $= -0.1481Wl$ when $a = \frac{l}{3}$ Max $y = \frac{-2W(l-a)^2a^3}{3EI(l+2a)^2}$ at $x = \frac{2al}{l+2a}$ if $a > \frac{l}{2}$; max possible value $= \frac{-Wl^3}{192EI}$ when $x = a = \frac{l}{2}$ $k = \frac{192EI}{l^3}$

4 x stiffer!

64 x stiffer!

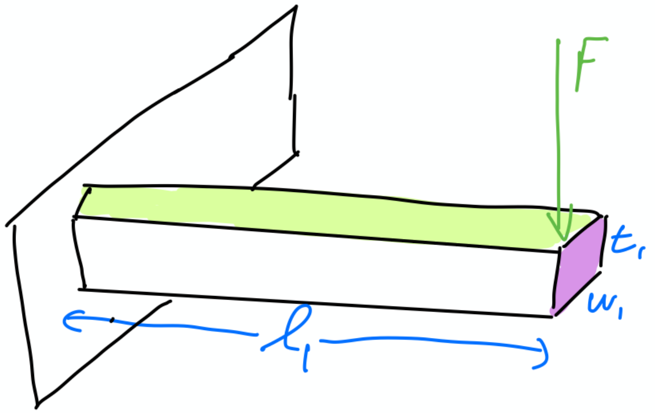
Resonance frequency depends on edge (support) conditions

Table 1: Fundamental Frequency *vs.* Geometry for *SiC*, [Si], and (GaAs) Mechanical R

Boundary Conditions	**Resonator Dimensions** ($L \times w \times t$, *in* μ*m*)			
	$100 \times 3 \times 0.1$	$10 \times 0.2 \times 0.1$	$1 \times 0.05 \times 0.05$	
Both Ends Clamped or Free	*120 KHz* [77] (42)	*12 MHz* [7.7] (4.2)	*590 MHz* [380] (205)	40x stiffness
Both Ends Pinned	*53 KHz* [34] (18)	*5.3 MHz* [3.4] (1.8)	*260 MHz* [170] (92)	8x stiffness
Cantilever	*19 KHz* [12] (6.5)	*1.9 MHz* [1.2] (0.65)	*93 MHz* [60] (32)	

“Nanoelectromechanical Systems”, M. L. Roukes, Technical Digest of the 2000 Solid-State Sensor and Actuator Workshop, Hilton Head Isl., 6/4-8/2000.

In bending

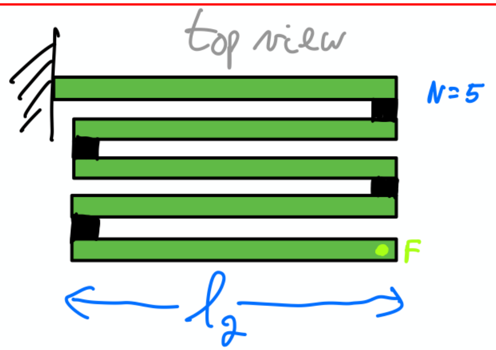


$$k_1 = \frac{3EI_1}{l_1^3}$$

$$I_1 = \frac{1}{12} w_1 t_1^3$$

$$k_1 = \frac{3EI_1}{l_1^3}$$

Single beam



$$k_2 = ??$$

Serpentine beam, pure bending

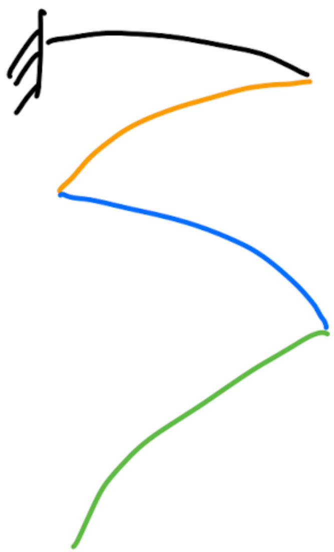
Springs in series?

How do they deform?

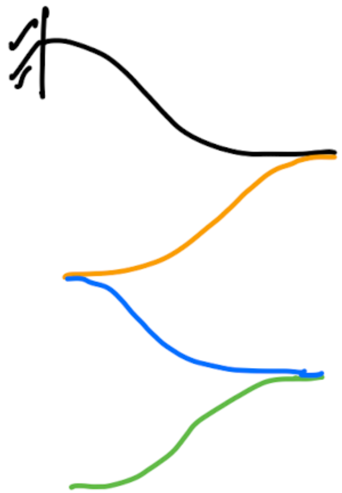
$$k_2 = k_1/N$$

?

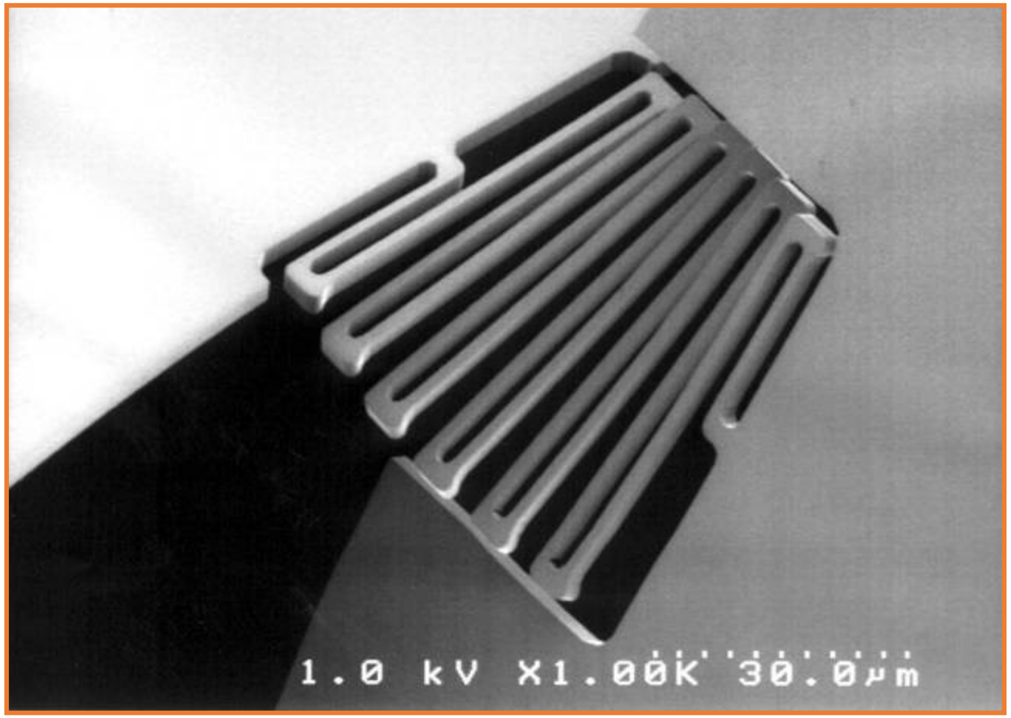
Serpentine beam. What assumptions to make about bending shape?



Bending angle keeps increasing at each end of the folded beams

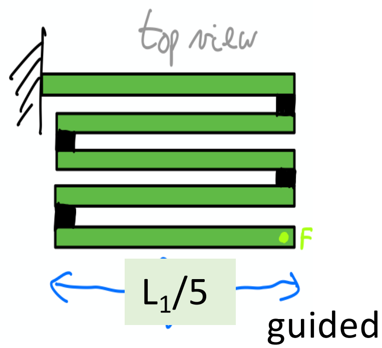


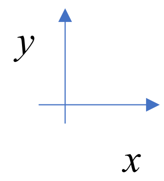
angle 0 at each end of the folded beams



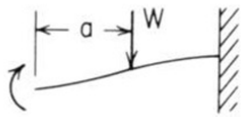
Problem 1. Spring design, one end fixed, one end guided

- We replace a single cantilever of length L_1 with a 5x shorter spring ($L_2 = L_1 /5$). We want the same total bending stiffness for out of plane motion (force in *z* direction)
- 1. Write an expression linking *w* and *t* and the number of serpentine segments *N*
- 2. Make 3 reasonable spring designs to get k=1 N/m for L_2=100 μm and material is Silicon (you find the values)





b. Left end guided, right end fixed



$$k_1 = \frac{12 E I_1}{l_1^3} = \frac{12}{12} E \frac{w_1 t_1^3}{l_1^3} = E \frac{w_1 t_1^3}{l_1^3}$$

$$k_2 = \frac{12 E I_2}{l_2^3} \frac{1}{N} \qquad l_2 = \frac{l_1}{5}$$

in series

$$k_2 = 12 \frac{E}{N} \frac{1}{12} \frac{w_2 t_2^3}{l_1^3} \cdot 5^3 = 125 \frac{E}{N} \frac{w_2 t_2^3}{l_1^3}$$

$$= E \frac{w_1 t_1^3}{l_1^3}$$

$$\boxed{w_2 t_2^3 = w_1 t_1^3 \cdot N \frac{1}{125}} \quad \boxed{\text{with } L_1 = 5 L_2}$$

$k = 1 N/m$ $\quad L_2 = 100\,\mu m$ $\quad E = 160\ GPa$

$N = 5$

$$k_2 = \frac{E w t^3}{L^3}\frac{1}{N}$$

$$\boxed{w t^3 = \frac{N k L^3}{E}}$$

$$t = \sqrt[3]{\frac{N k L^3}{E w}}$$

$$t = \sqrt[3]{\frac{1}{w}}\ \sqrt[3]{\frac{5.1.[100^{-4}]^3}{160.10^9}}$$

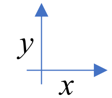
$$t = 3.10^{-8}.(w)^{-1/3} \quad \text{or} \quad w = \left[\frac{3.10^{-8}}{t}\right]^3$$

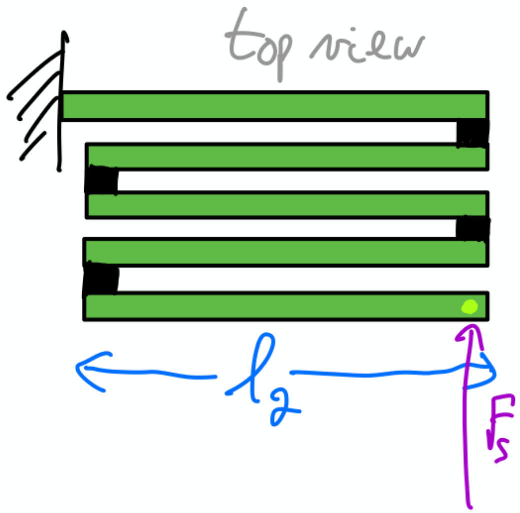
$$\boxed{\begin{array}{lll} w = 125\,\mu m & t = 0.6\,\mu m & (a) \\ w = 15\,\mu m & t = 1.2\,\mu m & (b) \\ w = 1\,\mu m & t = 3\,\mu m & (c) \end{array}}$$

L = 100 µm

- 4. Express the side bending stiffness for force in *y* direction vs. geometry

- 5. For the 3 reasonable spring designs that gave 1N/m in z direction, what stiffness do you get in the y direction?





4. Side Bending. still N springs in parallel.

but $I_{side} \neq I_{top}$

$$k = \frac{12 E I_2}{L^3} \frac{1}{N} = \frac{E}{N} \frac{w^3 t}{L^3}$$

$$= \frac{160 \cdot 10^9}{[100 \cdot 10^{-6}]^3} \cdot \frac{1}{5} \cdot (10^{-6})^4 \cdot w^3 t$$

in μm

$$= 3.2 \cdot 10^{-2} w^3 t$$

5.

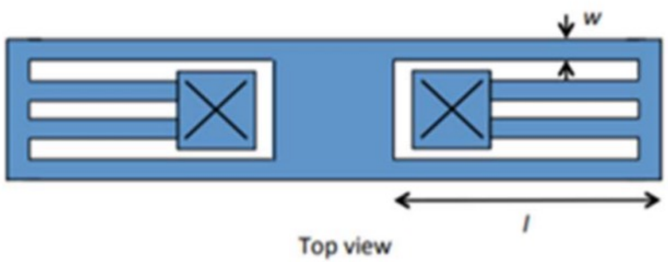
$$k = 37'000 \ N/m$$
$$= 130 \ N/m$$
$$= 0.1 \ N/m$$

$w = 125 \mu m$ $t = 0.6 \mu m$ (a)

$w = 15 \mu m$ $t = 1.2 \mu m$ (b)

$w = 1 \mu m$ $t = 3 \mu m$ (c)

design (a) offers much better lateral stiffness



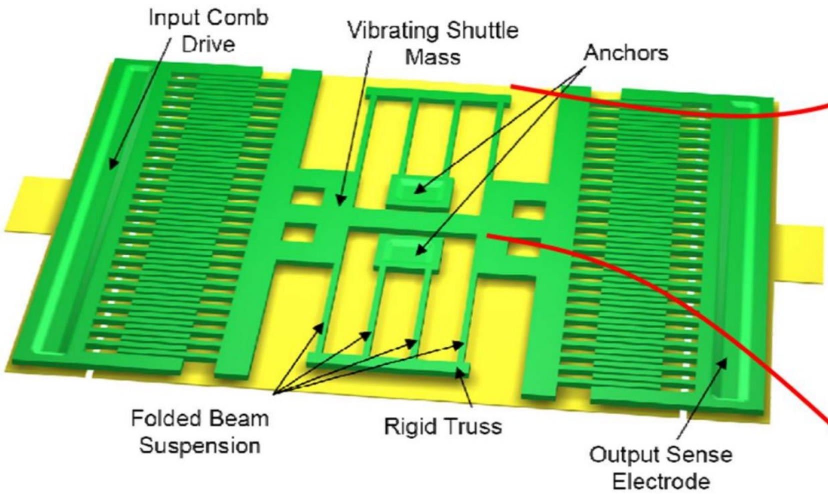
Folded flexure

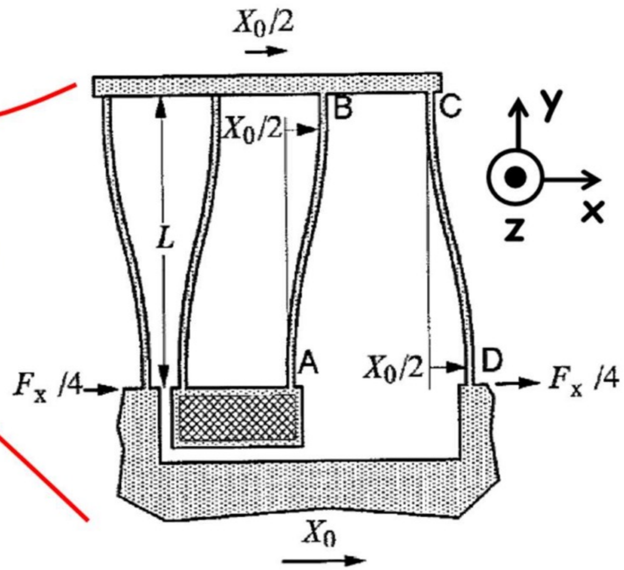
$$k_{spring} \approx 2Et\left(\frac{w}{l}\right)^3$$

for a very stiff truss

Equation for in-plane displacement;
for out-of-plane exchange *t* and *w*

See Henein Micro-200
to compute *k*





Oscillator, comb-drive, flexure suspension